Effects of temperature-dependence of viscosity and viscous dissipation on laminar flow heat transfer in circular tubes

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Heat transfer effects of variable viscosity and viscous dissipation for heated developing laminar flows in circular tubes have been investigated. Three studies are reported covering a comprehensive range of input data for the case of constant wall heat flux. Initially the program was used to predict the effect on heat transfer of temperature-dependent viscosity via a general temperature power relation. In addition, predictions were made for nine particular fluids covering a range of Prandtl numbers from 0.025 to 12 500, and a range of Brinkman numbers from 1.8×10^{-10} to 6.8×10^{3}. A more detailed study was made for two particular oils covering a range of practical interest. For the liquids considered their viscosity temperature-dependence resulted in enhancement of heat transfer, whereas for fluids with a Prandtl number <200 the effect of viscous dissipation was negligible, and for fluids of a Brinkman number $>2\times10^{-2}$ the outcome was a reduction of heat transfer. A numerical instability problem occurred for situations of very high viscous dissipation which limited the length of duct that could be examined.

Keywords: *laminar flow, heat transfer, viscosity*

Convective heat transfer data are of wide importance in the design of engineering equipment such as heat exchangers. Therefore, any physical effects which may lead to enhancement in heat transfer performance are worth investigating. At best, the consequence is a reduction in exchanger size for the same overall heat transfer. At least, however, the design can accommodate such physical effects more realistically. Certain experimental data for high viscosity liquids in laminar flow in tubes showed heat transfer enhancement (Butterworth and Hazell[1], and Martin and Fargie[2]). Two effects could be responsible for these changes in heat transfer, those of viscosity variation with temperature, and of viscous dissipation. In Refs 1 and 2 the enhancement was attributed, at least in part, to viscous dissipation. However, Collins[3] was able to show that the improvement was due to temperature variation of viscosity and that viscous dissipation tended slightly to reduce heat transfer. In the discussion of this paper[3], Martin recommended that more general data be obtained, and a comprehensive investigation was carried out. A wide range of parameters was covered, and the computer program developed by Collins[3] was used.

Three studies are reported. Firstly, a general temperature power relation was used, with generalized parameters, covering the extremes of interest of Reynolds and Prandtl numbers and heat flux. For each data run, three situations were studied: (1) constant viscosity, viscous dissipation ignored, (2) variable viscosity, viscous dissipation ignored, and (3) variable viscosity, viscous dissipation included.

Secondly, a representative range of liquids was studied, i.e. mercury, methyl chloride, freon, water, ethylene glycol, glycerin and several oils. Again, situations (1)–(3) were examined.

Finally, as the previous studies had shown an instability problem in the solutions for strong viscous dissipation effects, a more detailed study was carried out for a limited range of parameters. This included an accuracy check against data of Brinkman[4] and Ou and Cheng[5] for zero and finite wall heat flux, respectively. Also the study was designed to investigate the separate effects upon heating of the different viscous dissipation parameters.

Analysis of problem

The basic equations are derived for a two-dimensional flow field inside a circular tube. The cylindrical co-ordinates z' and r' represent axial distance from entry and radius from axis, respectively. Laminar, steady, axially symmetric flow was

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assumed. Eqs (1)–(4) represent the following principles, respectively: conservation of mass, conservation of momentum in the axial and radial directions, and the conservation of energy.

$$\frac{\partial v_r}{\partial r'}+\frac{v_r}{r'}+\frac{\partial v_z}{\partial z'}=0 \tag{1}$$

$$\rho\left(v_r\frac{\partial v_z}{\partial r'}+v_z\frac{\partial v_z}{\partial z'}\right) =-\frac{\partial p}{\partial z'}-S\rho g+\mu\left(\frac{\partial^2 v_z}{\partial z'^2}+\frac{1}{r'}\frac{\partial v_z}{\partial r'}+\frac{\partial^2 v_z}{\partial r'^2}\right) +2\frac{\partial \mu}{\partial z'}\cdot\frac{\partial v_z}{\partial z'}+\frac{\partial \mu}{\partial r'}\left(\frac{\partial v_r}{\partial z'}+\frac{\partial v_z}{\partial r'}\right) \tag{2}$$

$$\rho\left(v_r\frac{\partial v_r}{\partial r'}+v_z\frac{\partial v_r}{\partial z'}\right) =-\frac{\partial p}{\partial r'}+\mu\left(\frac{\partial^2 v_r}{\partial r'^2}+\frac{1}{r'}\frac{\partial v_r}{\partial r'}-\frac{v_r}{r'^2}+\frac{\partial^2 v_r}{\partial z'^2}\right) +2\frac{\partial \mu}{\partial r'}\cdot\frac{\partial v_r}{\partial r'}+\frac{\partial \mu}{\partial z'}\left(\frac{\partial v_r}{\partial z'}+\frac{\partial v_z}{\partial r'}\right) \tag{3}$$

$$v_r\frac{\partial T'}{\partial r'}+v_z\frac{\partial T'}{\partial z'}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T'}{\partial z'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}+\frac{\partial^2 T'}{\partial r'^2}\right)+\frac{\mu\phi}{\rho C_p} \tag{4}$$

The properties C_p, k and ρ are assumed constant, and S in the buoyancy term of Eq (2) is taken as zero in this forced convection investigation. Viscosity variation is fully allowed for and the treatment permits any expression which may be differentiated with respect to temperature.
Hence the term $\partial\mu/\partial r'$ in Eq (2) becomes for example:

$$\frac{\partial \mu}{\partial r'}=\frac{d\mu}{dT'}\frac{\partial T'}{\partial r'}$$

Allowing for up to three input constants for viscosity:

$$\mu=\mu_A\cdot f(C_1, C_2, C_3, T') \tag{5}$$

The viscous dissipation function ϕ is given by:

$$\phi=2\left\{\left(\frac{\partial v_r}{\partial r'}\right)^2+\left(\frac{v_r}{r'}\right)^2+\left(\frac{\partial v_z}{\partial z'}\right)^2\right\}+\left\{\frac{\partial v_z}{\partial r'}+\frac{\partial v_r}{\partial z'}\right\}^2 \tag{6}$$

For a numerical solution, the integral continuity equation is also used, ie:

$$\pi R^2 U_o=\int_0^R 2\pi r' v_z\,dr' \tag{7}$$

In this treatment the integral step-wise energy balance is used as part of the analysis instead of as a check. The technique, therefore, guarantees the

Notation

A	Wall area
Br	Constant heat flux Brinkman number $((Q_d/H_f)(Pr\,Re/2\,M))$
C_1, C_2, C_3	Constants defined by Eq (5)
C_p	Specific heat of fluid at constant pressure
f	Viscosity temperature function used in Eq (5) and defined for this work by Eq (9)
G	$(R^3 g/\nu_A^2)$
Gz	Graetz number $(2\,Pr\,Re/z)$
g	Acceleration due to gravity
k	Thermal conductivity of fluid
M	Number of radical positions in finite difference treatment
Nu	Nusselt number $(2QR/k_m A(T'_R - T'_M))$
P	Dimensionless pressure $((p-p_0)/\rho_m U_o^2)$
p	Pressure
Pr	Fluid Prandtl number $(\mu_A(C_p/k)_m)$
Q	Wall heat flow rate
R	Tube radius
R_h	Relative heating effect defined by Eq (12)
Re	Reynolds number $(2v_z R\rho_m/\mu_A)$
r	Dimensionless radial co-ordinate (r'/R)
r'	Radial co-ordinate
S	Natural convection parameter in Eq (2)
T	Dimensionless temperature $(G(T'-T'_o)/T'_o)$
T_α	Function defined by Eq (9)
T'	Temperature
U	Dimensionless axial velocity (v_z/U_o)
U_o	(Dimensional) entrance axial velocity (assumed slug flow)
V	Dimensionless radial velocity (v_r/U_0)
v	Velocity
z	Dimensionless co-ordinate (z'/R)
z'	Axial co-ordinate

Greek letters

ζ	$(z/Re\,Pr)$
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density
τ	$(T'/2\,Re\,Pr\,Q_d)$
ϕ	Viscous dissipation function defined by Eq (6)

Subscripts

A	property reference point, Eq (5)
m	mean value
M	local bulk mean value
R	wall condition
r, z	radial and axial directions
o	entrance condition

Finite difference parameters

H_f	Dimensionless wall heat flux $(QGR/AT'_o M k_m)$
Q_d	Dimensionless viscous dissipation parameter $(2GU_o^2/T'_o C_p Re)$

energy balance. Allowing for both a constant wall heat flux and viscous dissipation the equation is:

$$\pi R^2 \rho U_o C_p (\Delta T')_M = \frac{Q}{A}(2\pi R)\,\Delta z' + \int_0^R \mu\phi 2\pi r'\,dr'\,\Delta z' \qquad (8)$$

where $(\Delta T')_M$ is the increase in the mixed-mean temperature over a length $\Delta z'$.

The boundary conditions for the entry, the axis and the wall, respectively, are:

at $z'=0$, $v_z=U_o$, $v_r=0$, $p=p_o$, $T'=T'_o$ for all r'

at $r'=0$, $v_r=0$, $\frac{\partial v_z}{\partial r'}=0$, $\frac{\partial p}{\partial r'}=0$, $\frac{\partial T'}{\partial r'}=0$ for all z'

and

at $r'=R$, $v_r=0$, $v_z=0$ for all z'

As an alternative to the uniform velocity profile a fully-developed parabolic profile may be used. Also, any non-recirculating entry profile is feasible in principle.

These equations were made dimensionless in U, V, P, T, r and z, using the substitutions defined in the Notation. As the overall analysis and computer program accommodate natural convection effects, the parameter G is used involving the gravitational acceleration g. Strictly, in a forced convection analysis G is redundant because it affects every term in the energy equation identically. However, for the sake of consistency, the definitions are retained. The dimensionless equations were replaced by a set of implicit finite-difference equations at a given axial step, ie no unknown variable could be solved explicitly solely in terms of variables already solved at a previous step. The resulting set of linear equations for all radial positions at a given step were solved by simple Gaussian elimination. A marching procedure was then used in the axial direction. The essential techniques are discussed by Collins[3].

Finally, for the studies reported in this paper, the general viscosity–temperature relation used was:

$$\mu = \mu_A \cdot (T'/T'_A)^{-C_1} \equiv \mu_A \cdot T_\alpha \qquad (9)$$

Keynejad[6] showed that for a whole range of liquids of interest, Eq (9) was valid, with values of C_1 up to about 3.

Results for generalized liquids

These were defined as having $Pr_0 = 1000$ and four values of C_1 from 0 to 3. Runs were specified by permuting low and high values of Reynolds numbers with extreme values of heat flux. Quantitative results are omitted because of lack of space; these (*Nu* values) were originally studied basically as a function of *Gz*. The first set of runs ignored viscous dissipation effects. For $C_1 = 0.0$ there is no viscosity variation and *Nu* values are independent of heat flux. At any *Gz*, with $C_1 \neq 0$, there is a consistent increase in *Nu*. If the heat flux is increased there is a further increase in *Nu*.

When viscous dissipation is accounted for, most runs show a slight reduction in *Nu* at all *Gz*. This is consistent with results reported previously[3]. Some runs, however, showed an inconsistent increase in *Nu*.

Finally, a standard heat transfer result is that for constant properties, *Nu* at a given *Gz* is independent of *Re*. It was also found here that for a given C_1 and heat flux, *Nu* is independent of *Re* at constant *Gz*.

The viscous dissipation results displayed some ambiguity. Also it became apparent that an inordinate amount of computation would be needed to generalize all parameters. For these reasons, it was decided to make predictions for a typical selection of specific liquids. In practice it was found that high *Pr*, high C_1 and high viscous dissipation tended to go together, thus justifying the approach.

Results for specific liquids

Nine liquids were chosen giving a range of Pr_0 and C_1 as in Table 1; the *Br* values indicate the likely relative importance of viscous dissipation. Common input data were as follows: $R = 0.01$ m, to maximize viscous dissipation, inlet temperature = 20 °C, Re = 500 and 2500, and heat flux = 480 and 7200 W/m^2. The latter gave low and high heating rates with a ratio of 15 between them. All dimensionless input data are tabulated in Ref. 6. For each run, three situations were predicted (1) constant viscosity, viscous dissipation ignored (2) variable viscosity, viscous dissipation ignored, and (3) variable viscosity,

Table 1 Properties of liquids

Liquid	Pr_0	C_1	*Br*	
			Re = 500*	*Re* = 2500†
Mercury	0.0249	0.13342	1.803×10^{-10}	6.760×10^{-8}
Methyl chloride	2.63	0.1437	2.058×10^{-10}	7.713×10^{-8}
Freon	3.5	0.15838	9.220×10^{-11}	3.458×10^{-8}
Water	7.02	0.7835	9.120×10^{-9}	3.420×10^{-6}
Ethylene glycol	204	1.36667	7.019×10^{-5}	2.632×10^{-2}
Oil A	430	1.491	2.188×10^{-4}	8.203×10^{-2}
Oil B	2913	2.106	1.872×10^{-2}	7.021
Oil C	10 400	2.300	5.804	2.171×10^{3}
Glycerin	12 500	2.414	1.830×10^{1}	6.814×10^{3}

* *High* Q.
† *Low* Q.

viscous dissipation included. Situation (1) heat transfer for all runs is shown in Fig 1, which indicates the wide range of Gz covered.

The predictions converge at low Gz to the fully developed value of 4.36. Differences upstream are due to the uniform inlet velocity used for this study. The Gz range for each liquid is defined by a length of $2\frac{1}{2}$–400 diameters.

Detailed results are summarized in Tables 2–4 which show that the effect of viscosity variation with temperature for the low Pr liquids is up to about 8% for high heat fluxes (ignoring the initial mercury value) but ≤1.5% for low heat fluxes. With the other liquids (essentially oils) the effect is significant unless the heat flux is low. The effect of viscous dissipation may be ignored for a liquid of $Pr_0 < 200$ and only becomes important for $Br \sim \geq 2 \times 10^{-2}$, see Table 1 in conjunction with Tables 2–4. This value

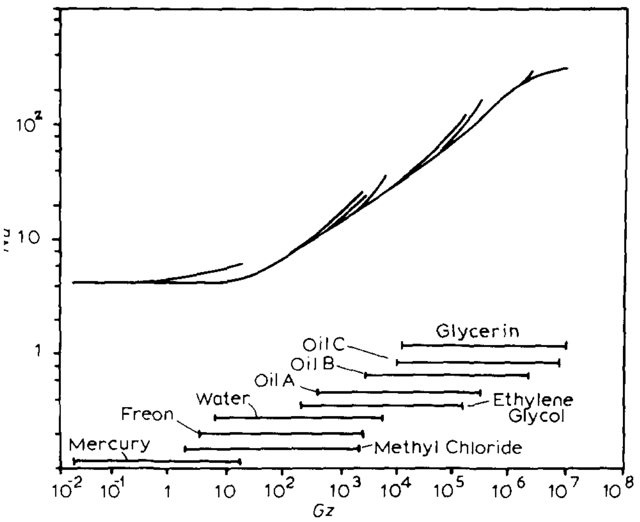

Fig 1 Constant property heat transfer

is consistent with the analysis of Ou and Cheng[5] whose graphical results give a minimum Br of 10^{-2} as noticeably affecting Nu.

Finally, with high viscous dissipation relative to the wall heat flux, unreliable results occur. These were diagnosed as due to a coupling problem affecting the solution of the energy equation when the predicted velocity profile became flattened. This is discussed in the Appendix. Despite some effort it was not found possible to overcome this.

Parameters affecting viscous dissipation

The flow field–energy coupling problem has two consequences. Firstly, the accuracy of the analysis needs to be reviewed. Secondly, it means that high viscous dissipation/variable viscosity flows, treated for example by Ockenden[8], cannot at present be carried out. As far as accuracy is concerned (see Appendix) the analyses of Brinkman[4], and Ou and Cheng[5] for constant properties and a fully-developed velocity field form additional checks on the solution of the energy equation only. The other consequence is not too severe. Even for flows which result in flattened velocity profiles, a developing field solution is still valid up to the inception of the flattening. These points are examined here.

Temperature fields in adiabatic flow

Brinkman's analysis of viscous dissipation[4] gives a universal dimensionless temperature field for an insulated wall. Fig 2, a comparison of his and present predictions, shows that agreement is good, particularly as it involves the solution of an entire two-dimensional axial–radial field.

Table 2 Heat transfer effects for liquids of low *Pr*

Liquid	Re	Gz	Low heat flux		High heat flux	
			Effect of viscosity variation (1)→(2) %	Effect of viscous dissipation (2)→(3)	(1)→(2) %	(2)→(3)
Mercury	500	4.978	+0.03	Nil	−17.94	Nil
		3.056×10^{-2}	+0.002		+8.47	
	2500	2.490×10	+0.01		−4.02	
		1.553×10^{-1}	+0.01		+1.74	
Methyl chloride	500	5.260×10^{2}	+0.23	Nil	+1.67	Nil
		3.285	+0.32		+0.48	
	2500	2.631×10^{3}	+0.11		+1.21	
		1.642×10	+1.11		+1.66	
Freon	500	7.0×10^{2}	+0.47	Nil	+2.85	Nil
		4.380	+0.59		+0.65	
	2500	3.5×10^{3}	+0.23		+2.17	
		2.190×10	+1.27		+2.31	
Water	500	1.404×10^{3}	+0.27	Nil	+3.61	Nil
		8.773	+1.50		+4.52	
	2500	7.019×10^{3}	+0.12		+1.87	
		4.393×10	+1.41		+8.32	

Table 3 Heat transfer effects for liquids of medium *Pr*

Liquid	*Re*	*Gz*	Low heat flux (1)→(2) (see Table 2) %	Low heat flux (2)→(3) (see Table 2)	High heat flux (1)→(2) %	High heat flux (2)→(3)
Ethylene glycol	500	4.080×10^4	+0.29	Negligible	+5.03	Negligible
		2.546×10^2	+4.03		+24.11	
	2500	2.040×10^5	+0.29		+4.39	
		1.275×10^3	+2.73		+22.78	
Oil A	500	8.591×10^4	+0.50	Increase less than 0.5%	+9.13	Negligible
		5.373×10^2	+7.14		+36.57	
	2500	4.296×10^5	+0.62		+8.78	
		2.685×10^3	+4.81		+34.74	

Table 4 Heat transfer effects for liquids of high *Pr*

Liquid	*Re*	*Gz*	Low heat flux (1)→(2) (see Table 2) %	Low heat flux (2)→(3) (see Table 2)	High heat flux (1)→(2) %	High heat flux (2)→(3) %
Oil B	500	5.827×10^5	+1.64	Unstable predictions	+11.97	−0.03
		3.641×10^3	+7.90		+47.86	−0.08
	2500	2.913×10^6	+1.00		+4.49	−1.03
		1.821×10^4	+4.75		+35.69	−12.24
Oil C	500	2.080×10^6	+0.09	Unstable predictions	+5.44	+0.75
		1.300×10^4	+2.00		+37.43	−74.87
	2500	1.040×10^7	+0.09		+1.13	−64.44
		6.500×10^4	+2.00		+23.01	—
Glycerin	500	2.500×10^6	+0.19	Unstable predictions	+2.57	−1.41
		1.563×10^4	+2.01		+22.34	−75.74
	2500	1.250×10^7	+0.04		+0.53	−84.81
		7.812×10^4	+0.98		+12.51	—

Viscous dissipation relation

If the step-wise energy balance equation, Eq (8), is made dimensionless, using the definitions in the Notation, the following form results:

$$\tfrac{1}{2}(\Delta T)_M = \frac{2H_f}{Re\,Pr} M\,\Delta z + \int_0^1 \phi_{ND}\, r\,dr\,\Delta z \tag{10}$$

where

$$\phi_{ND} = Q_d \times T_\alpha \times \left\{2\left[\left(\frac{\partial V}{\partial r}\right)^2 + \left(\frac{V}{r}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2\right] + \left(\frac{\partial V}{\partial z} + \frac{\partial U}{\partial r}\right)^2\right\} \tag{11}$$

Hence, the *relative* viscous dissipation effect upon T_M is given by the ratio of the second term to the first on the RHS of Eq (10):

$$\text{ie relative heating effect} = \frac{\int_0^1 \phi_{ND}\, r\,dr\,\Delta z}{\dfrac{2H_f}{Re\,Pr} M\,\Delta z}$$

or

$$R_h = Br \times \int_0^1 T_\alpha \times V_f\,dr \tag{12}$$

where Br is the customary Brinkman Number and V_f is a 'velocity field function'. Although this relative heating effect is strictly only for T_M, it is shown later to apply in principle to the whole solution. The relation in Eq (12) implies three situations. Firstly, if the viscosity is constant ($T_\alpha = 1$) and the velocity field parabolic, R_h is then a function of Br only. Secondly, and admittedly not consistently, μ can be

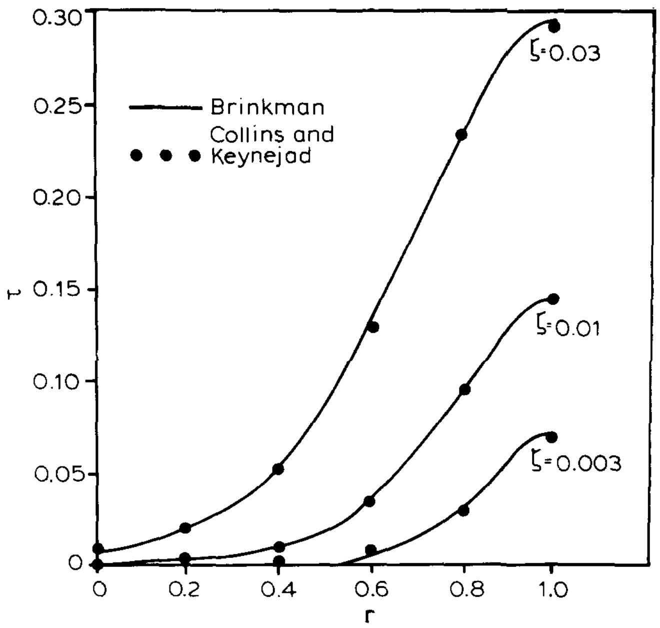

Fig 2 Temperature field in adiabatic flow

allowed to vary in the energy Eq (4) only. The velocity field is then still parabolic, and R_h will also reduce with T_α. Finally, and consistently, μ can vary fully and thus affect the coupled velocity field. R_h is then a complex function given fully by Eq (12).

The effect on the Nusselt number arises from its being inversely proportional to $(T_R - T_M)$. For the first situation the viscous dissipation will tend to heat the wall region more than the centre. Hence $(T_R - T_M)$ increases and Nu reduces. The actual reduction depends on Br, which is the relative value of viscous dissipation to wall heating. For higher heat fluxes, therefore, viscous heating is of less significance. For a very low heat flux, the problem approximates to adiabatic flow with viscous dissipation. Nu then becomes very small. These considerations (also discussed by Shah and London[7]) are not necessarily invalidated by allowing μ and V_f to vary. For μ varying, but V_f constant, the effect reduces more near the wall than the centre. This is because of the temperature distribution. If V_f varies, it implies an acceleration near the wall which flattens the velocity profile. This concentrates the heating effect in the wall region because the dominant velocity gradient is $(\partial U/\partial r)$.

Effect of *Br* only

This situation has been treated by Ou and Cheng[5], who assumed constant μ and a parabolic profile. Predictions of Nu are compared in Table 5.

Agreement is very good except at $\zeta = 10^{-3}$. However, consideration of the data of Shah and London shows that probably the values of Ou and Cheng are less reliable at that length. The results confirm the accuracy of the present treatment for constant μ. They also demonstrate the consistent reduction in Nu as Br increases.

Detailed parametric study

The parameters in Eq (12) are now examined in detail. To relate this final study to other results the work includes runs carried out previously. These cover the practical range of interest of Br (Table 4) from where its effect is very small (Oil B, $Re = 500$) to the numerical instability problem (Oil C). This gave four basic runs, each with the seven schemes defined in Table 6.

Table 6 Schemes for detailed study of oils B and C (high heat flux)

Scheme	Viscosity variation	Q_d allowed for	H_f allowed for
A	Constant	√	×
B	Constant	×	√
C	Constant	√	√
D	In energy equation	√	√
E	Full	√	×
F*	Full	×	√
G*	Full	√	√

**Already run for Table 4*

Values of T_M and T_R are given for the entire range in Table 7 in order of increasing Br. For each sub-Table the variation of T_M with distance in scheme A is not linear because the velocity profile is developing. By comparing values of T_M for schemes A and B, the relative heating effect expressed by Br is clearly shown. Whereas in Table 7(a) the viscous heating is one order less than the wall heat flux, by Table 7(d) it has become three orders greater. Also, for any run and at any length the T_M's for schemes A and B add to give the T_M values for scheme C. This would be expected, as the numerical treatment enforces agreement of Eq (8). However, it is *also* true of T_R, a more significant effect, implying that (for constant properties) the viscous heating solution and wall heat flux solution are entirely additive. Ou and Cheng's relation for T (Eq (14), Ref 5) may similarly be separated into wall heat flux and viscous heating terms which are additive. Scheme D (see Table 6) means that only the viscous heating effect is affected by the variation in μ. Hence, for Table 7(a) schemes C and D are very similar because viscous dissipation is small, but from Table 7(a) to (d) there is a stronger reduction in both T's. Turning now to viscosity *and* flow-field variation, schemes E and A are comparable. The effect

Table 5 *Nu* comparison with data of Ou and Cheng[5]

Br \ Length ζ	10^{-3}		10^{-2}		10^{-1}	
	Ou and Cheng	This work	Ou and Cheng	This work	Ou and Cheng	This work
0	12.51	15.948 15.813*	7.53	7.529 7.494*	4.54	4.529 4.514*
0.01	12.28	15.801	7.41	7.412	4.45	4.435
0.1	11.24	14.590	6.49	6.499	3.71	3.738
0.5	8.0	10.885	4.17	4.201	2.22	2.200
1.0	5.79	8.26	2.90	2.913	1.39	1.453
2.0	3.80	5.575	1.81	1.806	0.93	0.87

* From Shah and London[7], p. 127

Table 7 Values of T_M/T_R

z / Gz \ Scheme	A	B	C	D	E	F	G
(a) Oil B, $Re=500$, $Br=1.872\times10^{-2}$							
5	0.0303	0.3339	0.3642	0.3639	0.0303	0.3339	0.3586
5.826×10^5	0.68	287.35	288.03	287.89	0.68	257.97	258.07
50	0.2541	3.3390	3.5931	3.5760	0.2535	3.3390	3.5423
5.826×10^4	6.49	926.57	933.06	930.04	6.50	760.71	764.08
800	3.9424	53.4242	57.3676	56.3556	3.8530	53.4242	55.3762
3.641×10^3	41.50	2563.33	2604.83	2575.50	40.75	1749.70	1765.06
(b) Oil B, $Re=2500$, $Br=0.4681$							
5	0.2297	0.0668	0.2965	0.2952	0.2289	0.0668	0.2290
2.913×10^6	7.74	170.38	178.12	177.83	7.81	164.01	165.88
50	1.4959	0.6678	2.1637	2.1041	1.4678	0.6678	1.9110
2.913×10^5	50.59	428.61	479.20	462.74	53.18	381.18	404.65
800	19.9679	10.6849	30.6527	27.2374	17.4312	10.6849	23.8008
1.821×10^4	374.04	1466.34	1840.38	1610.13	346.59	1084.25	1247.11
(c) Oil C, $Re=500$, $Br=5.804$							
5	0.02941	0.00105	0.03046	0.02990	0.02885	0.00105	0.02536
2.080×10^6	0.623	2.154	2.776	2.716	0.715	2.043	2.053
50	0.24688	0.01047	0.25735	0.22831	0.20860	0.01047	0.20244
2.080×10^5	8.692	6.460	15.151	10.538	9.867	5.658	10.268
800	3.83147	0.16746	3.99893	2.34689	1.60251	0.16746	1.59585
1.300×10^4	63.053	18.474	81.528	30.732	30.545	13.488	30.740
(d) Oil C, $Re=2500$, $Br=144.8$							
5	0.2227	0.0002	0.2229	0.1860	0.1769	0.0002	0.1299
1.040×10^7	7.499	1.711	9.210	6.275	8.9675	1.692	5.417
50	1.4502	0.0021	1.4523	0.9010	—	0.0021	0.7540
1.040×10^6	54.753	2.866	57.619	12.917	—	2.640	29.981
800	19.3575	0.0335	19.3910	6.4191	—	0.0335	—
6.500×10^4	560.528	10.690	571.219	49.708	—	8.697	—

is always to reduce T_M, but T_R's in scheme E appear initially higher before becoming less than for scheme A. Finally, scheme E of Table 7(d) has encountered the instability problem at a short distance from entry ($z=17.5$). For wall heat flux only, schemes F and B are comparable. For all results T_M is the same because of the energy balance, but T_R is always less for scheme F. As expected, for the same fluid and heat flux, the reduction is more marked for lower *Re*.

Scheme *G* is the total solution, and relative to scheme *F*, consistently shows the effect of viscous dissipation is to increase both T_M and T_R. The latter, however, is increased by a much larger amount. If scheme C is compared with A, D with C, and G with D it is possible to quantify the relative effects of *Br*, T_α and V_f in Eq (12). For Oil B, that is *low* viscous dissipation, the value of *Br* dominates the solution. This is because changes A to C are the most significant. For Oil C, the most significant change is C to D, ie the viscosity variation.

Heat transfer results

The resulting heat transfer is shown in Figs 3 and 4 for Oils B and C, respectively. In Fig 3(a), the effect of μ variation is beneficial and substantial. However, the effect of viscous dissipation is indiscernible; in fact it gives a slight reduction in *Nu* of up to −1.5% (μ constant) and −0.8% (μ varying).

Fig 3(b) shows about the same change due to μ variation; however, with the much higher *Br* there

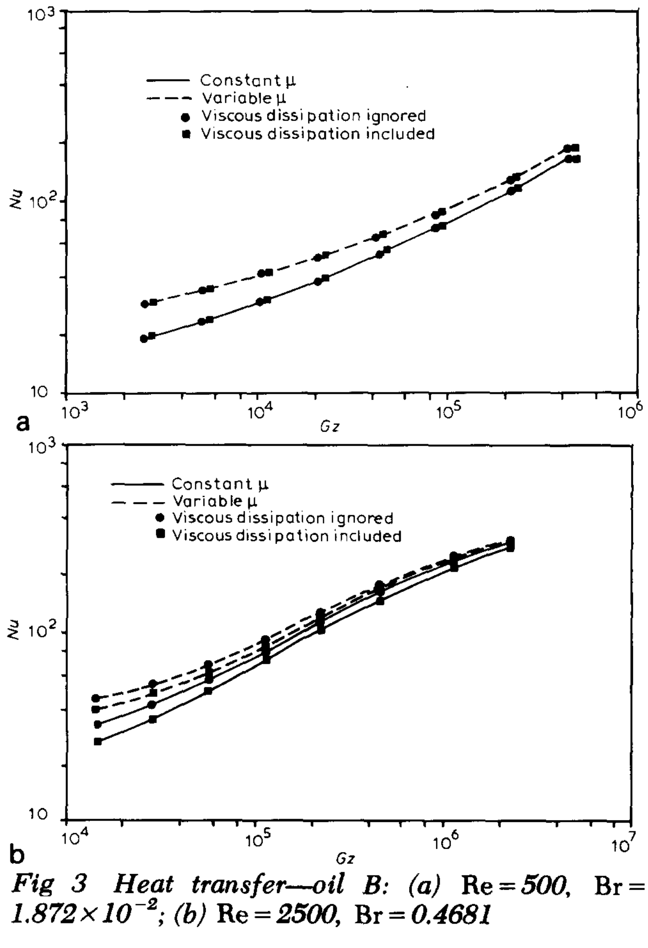

Fig 3 Heat transfer—oil B: (a) Re = *500,* Br = *1.872*×*10*$^{-2}$*; (b)* Re = *2500,* Br = *0.4681*

is substantial reduction due to viscous dissipation, up to −19.6% (μ constant) and up to −12.2% (μ varying). Fig 4(a) shows a continuation of the trend with *Br*. Again the reduction because of viscous dissipation is ameliorated by the variation of μ from 76.2% maximum to 54.5% maximum. Fig 4(b) is an extreme case with *Nu* (for constant μ and viscous dissipation included) decreasing to <1 and the full solution terminating because of numerical instability. However, all these results confirm that the variation of μ is beneficial in both increasing *Nu* in its own right, and in reducing the negative effect of viscous dissipation.

Flow-field results

The corresponding development of centre-line axial velocity is given in Figs 5 and 6. The effect of viscosity variation is greater for lower *Re* because of the lower velocities for the same wall heat flux. Also, the effect of viscous dissipation consistently increases with *Br*. Finally, the *U* and *V* profiles are given in Fig 7 for the last stable axial solution of Fig 6(b). It occurs immediately before the predicted appearance of a point of inflexion near the wall. The instability is disappointing, for the problem represented by this range of parameters is of considerable interest in non-Newtonian flows (Ref 8). This is confirmed by the *U*-profile, which is very flat compared with the fully developed parabolic shape. There is a compensating radially outward flow of fluid.

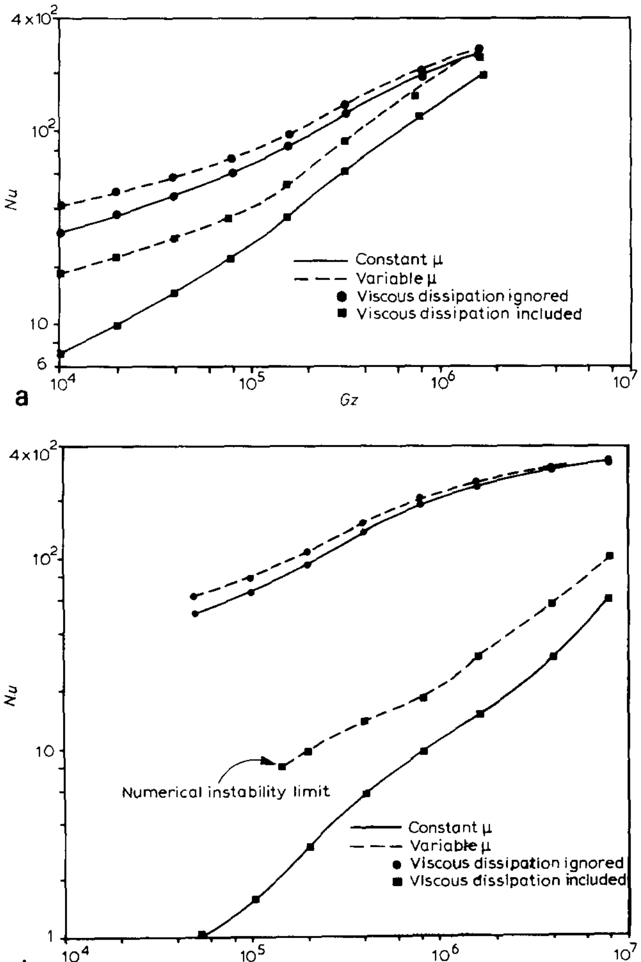

Fig 4 Heat transfer—oil C: (a) Re = *500,* Br = *5.804;* (b) Re = *2500,* Br = *144.8*

Generality of the analysis

This study is for constant wall heat flux for heated flows, a case of general engineering interest treated by Shah and London[7]. Experimental data for high *Pr* fluids were obtained by Butterworth and Hazell[1] (water–glycerol mixture) Martin and Fargie[2] (Oil B of this work), and Allen[9,10] (Oil A of this work). The latter data specifically relate to oil-cooled transformers. However, the constant wall temperature boundary condition, both for heated and cooled flows, is also of major engineering significance. It is similarly treated by Shah and London[7].

It is noteworthy, therefore, that the analysis and its program reported here are of wide application. It can allow for natural convection effects[9]. Also it deals equally well with uniform wall temperature (see Ref 11 where the method is fully described). It has been applied to a non-uniform wall temperature for annuli[12] in a solar power situation. Further, heat transfer with non-Newtonian fluids is of considerable current interest. Joshi and Bergles[13] recently developed a less general method allowing for variable viscosity and power-law shear stress effects with constant heat flux. They recommended the development of a similar method for constant temperature[18]. This is being carried out and initial results are encouraging.

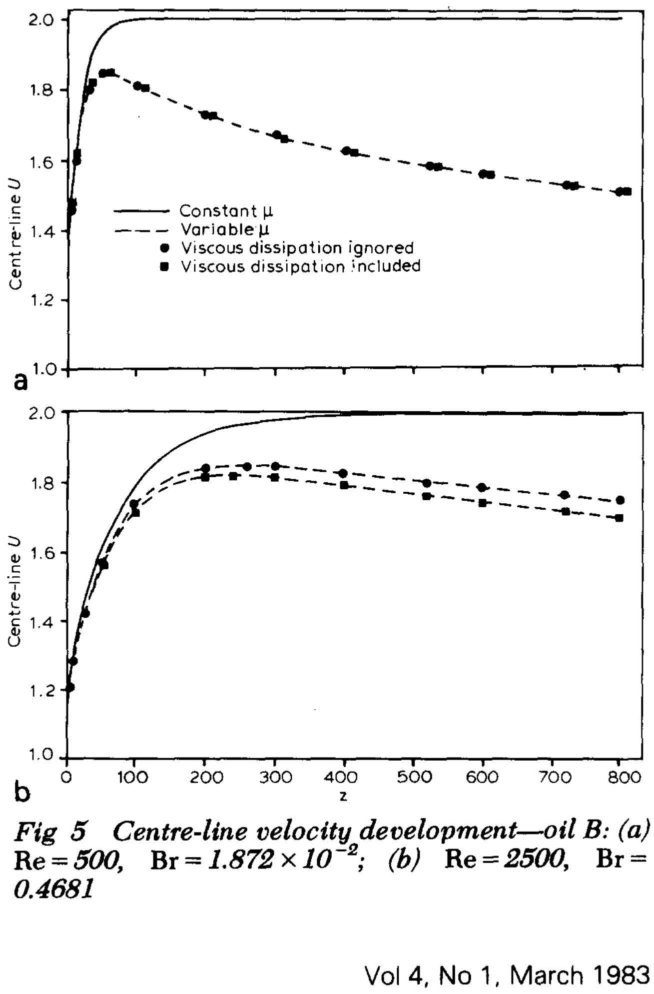

Fig 5 Centre-line velocity development—oil B: (a) Re = *500,* Br = *1.872* × *10*$^{-2}$*; (b)* Re = *2500,* Br = *0.4681*

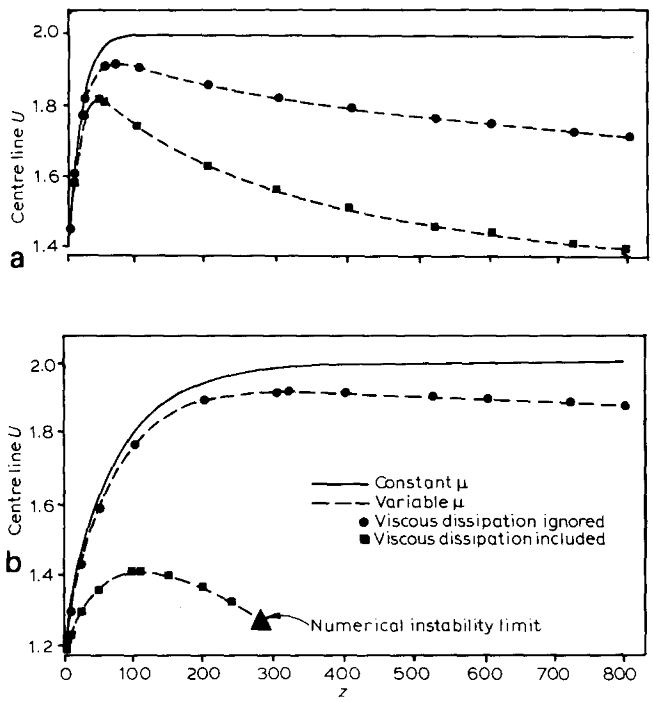

Fig 6 Centre-line velocity development—oil C (a) Re = *500*, Br = *5.804*; *(b)* Re = *2500*, Br = *144.8*

In work not yet published, a study has been made of Richardson's analysis for extended Lévêque solutions[14]. This analysis was originally thought to be valid for high Gz only. It has been posible to show, however, that by accounting correctly for the local bulk mean temperature, Richardson's analysis is now valid for much lower Gz[15]. This conclusion has practical implications for analysis of more complex problems.

Conclusions

An investigation has been made of the effects on heat transfer of allowing for temperature dependence of viscosity and viscous dissipation. The first effect consistently increases Nu, the increase being larger for higher heat fluxes and higher viscosity–temperature coefficients. It is negligible at low Pr.

Viscous dissipation is negligible for $Pr_0 < 200$. The constant heat flux Brinkman number is a reliable criterion, and viscous dissipation becomes apparent at $Br > 2 \times 10^{-2}$. For very high Br, a numerical instability problem limited the length of duct which could be studied.

In further work it is intended to re-examine the instability problem and to obtain data for constant wall temperature. Also non-Newtonian fluid characteristics are being incorporated into the analysis.

Acknowledgement

Computational facilities were provided by the University of London Computer Centre.

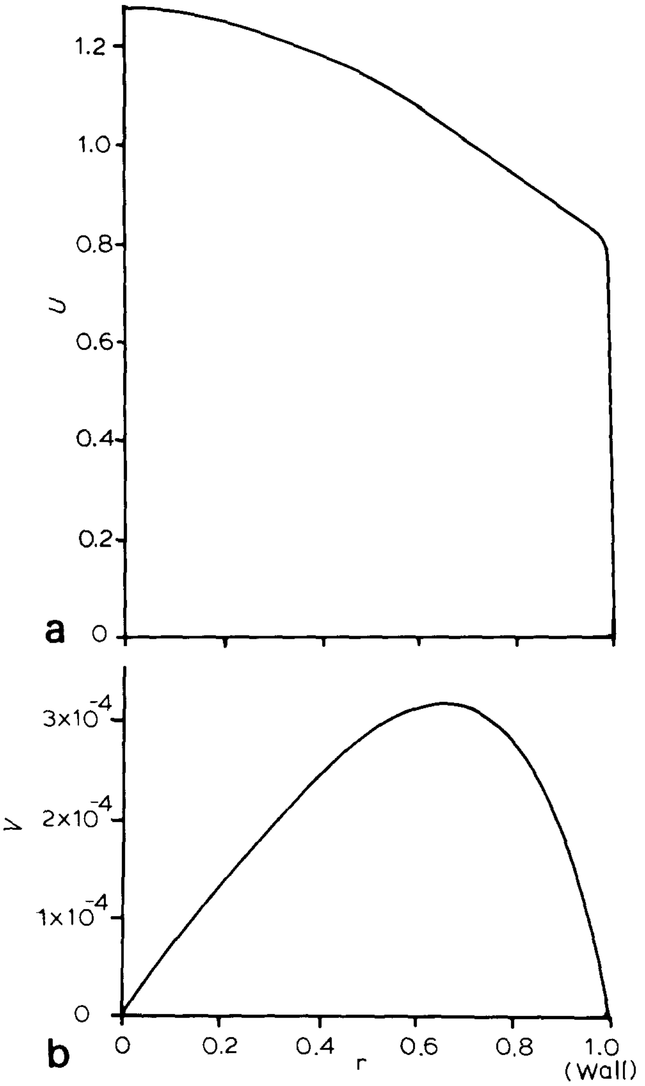

Fig 7 Flow field at stability limit of Fig 6(b): (a) U*-profile; (b)* V*-profile*

Appendix

The numerical instability problem

The current numerical treament (see Collins[11]) involves solving, at a given axial step, the finite-difference equations for the U, V and P's from Eqs (1)–(3) and (7). The solved velocity field is then substituted into finite-difference equivalents of Eqs (4) and (8), to give a solution for the T's. To check that coupling due to property variation between the two sets of equations is properly converged, a semi-iteration is carried out at the same step. The solved T's are used in the U, V, P equations, and a final solution made for the T equations. Only then does the solution march to the next axial step.

Numerical stability in the solution involves the general stability of the basic equations, and several coupling effects. These are: (a) in the momentum equations due to variation of μ and ρ (not considered here); (b) in the energy equation due to the velocity field (viscous dissipation ignored), and (c) in the energy equation due to both μ variation and the velocity field in the viscous dissipation term.

The general stability of the basic equations has been studied (Appendix II, Ref 11) and is 'very satisfactory'. The coupling effects (a) and (b) have been demonstrated always to converge by means of the semi-iterative check. The accuracy has been checked (Collins[16]) by many comparisons with analysis and experiment.

Here, therefore, only coupling (c) is considered, as the current solutions of the U, V, P equations never displayed instability. Further, it is the coupling and not the basic treatment which causes the problem. This is because the comparison with data of Brinkman[4], and Ou and Cheng[5] is satisfactory for a fixed velocity field and constant μ. In this work the instability only arises for a flat velocity profile, and not for the μ variation itself. This is conclusively confirmed by the comparison in Ref 3 with experimental data of Polak, and Hersey and Zimmer, and the analysis of Martin[17] for unheated high viscous dissipation flows. There the same problem arose near entry to a duct where the velocity profile was again flattened. For the rest of the flow, where of course μ and the velocity field were varying, no such problem arose. The overall comparison favoured the current treatment.

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